Long-lived memory in sliding spin chains

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We study a system of two ferromagnetic one-dimensional Ising chains coupled to a thermal bath, which are driven out of equilibrium by being moved past one another at a constant speed. We show that even at modest speeds, magnetic friction between the two chains significantly increases the ability of the system to order. In particular, at inverse temperature β, Ising coupling J, and sliding speed v, the dynamics retains memory of its initial magnetization for a time that increases from $\exp(O(\beta J))$ at $v=0$ to $\exp\big(O((\beta J)^2 v)\big)$ when $v > v_c$, where v_c is a small constant. Magnetic friction thus provides a simple mechanism for parametrically slowing down thermalization in a one-dimensional magnet.

Ordered phases of matter retain memory of their initial conditions: prepare a ferromagnet with a particular magnetization, and at low enough temperatures, its magnetization will remember its initial value for thermodynamically long times. Historically, order has been understood through the lens of spontaneous symmetry breaking. However, approaching order from the perspective of memory is better suited for understanding *non*equilibrium phases of matter, some of which are capable of retaining memory in the absence of *any* symmetry [1–5]. Given the outstanding difficulties in classifying nonequilibrium phases of matter, and the immense technological importance of magnetic memories [6], understanding how to construct physical systems that are good at remembering information is a matter of both fundamental and applied importance. While it is an old subject [1–5], it has also seen several recent advances [7–11].

Memory in one dimension, where equilibrium systems cannot order, is both especially interesting, and especially poorly understood. Gács (in)famously constructed an example of a 1d cellular automaton that retains memory for thermodynamically long times in the absence of any symmetry [4], but his construction—based on an infinite hierarchy of self-simulating Turing machines—is so complicated that it has yet to be independently understood. Attempts to construct simpler models have so far come short [3, 12, 13]. An outstanding question thus remains: are there *simple* ways for 1d systems to retain memory? Or, failing this, is there at least a simple model which gets close?

This work addresses the last question by constructing a 1d spin system that, when driven out of equilibrium in a simple way, remembers its initial conditions for a very long time, even in the presence of noise. While this time is finite in the thermodynamic limit, it is nevertheless significantly longer than any 1d equilibrium system can manage, in a sense made precise below.

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The construction in our paper is a spin system that retains a single bit of memory in the sign of the average magnetization m. In this setting, we define the *memory time* t_{mem} as the expected time for a maximally-magnetized state to lose memory of its initial magnetization:

$$t_{\mathsf{mem}} = \mathbb{E} \min_{s=\pm 1} \{t \,:\, |m(t)| < 1/2\,,\, m(0) = s\} \qquad (1)$$

where the constant $1/2$ is chosen rather arbitrarily. t_{mem} can be viewed as a proxy for the mixing time, for which it sets a lower bound [14].

To orient ourselves about the behavior of t_{mem}, consider Glauber dynamics run at temperature T for the 1d Ising chain with coupling J. Once a minority domain (of any size) is formed, its endpoints will execute unbiased random walks. Since the energy cost to nucleate a minority domain is $\Delta E = 4J$, it follows that $t_{\mathsf{mem}} \sim e^{4\beta J}$ [15]. More generally, for any 1d spin system undergoing Glauber dynamics with respect to a local Hamiltonian H, a standard Arrhenius-type argument shows that $t_{\mathsf{mem}} \lesssim e^{O(\beta J)}$, where J is the largest local energy scale in H [16, 17]. To get a longer memory time than this, we must go out of equilibrium.

Sliding Ising models: In this paper, we show that subjecting a ferromagnetic 1d Ising model on a two-leg ladder to a very simple type of nonequilibrium drive can significantly increase t_{mem}. We drive the system out of equilibrium by sliding the two legs of the ladder past one another at velocities $\pm v$, as illustrated in Fig. 1 **a**. More precisely, letting $s_i^{(\mathsf{t}/\mathsf{b})} \in \{\pm 1\}$ be the ith spin on the top / bottom leg of the ladder, we consider a system governed by the time-dependent Hamiltonian [18]

$$H(t) = -J \sum_{i;a\in\{\mathsf{t},\mathsf{b}\}} s_i^{(a)} s_{i+1}^{(a)} - J \sum_i s^{(\mathsf{t})}_{\lfloor i-vt\rfloor} s^{(\mathsf{b})}_{\lceil i+vt\rceil}. \qquad (2)$$

We use updates where a configuration C is replaced by a configuration C' according to a rate $w(C\to C')$ set by Glauber dynamics at temperature T:

$$w(C\to C') = \gamma(1 + e^{\beta(E_{C'}(t) - E_C(t))})^{-1}, \qquad (3)$$

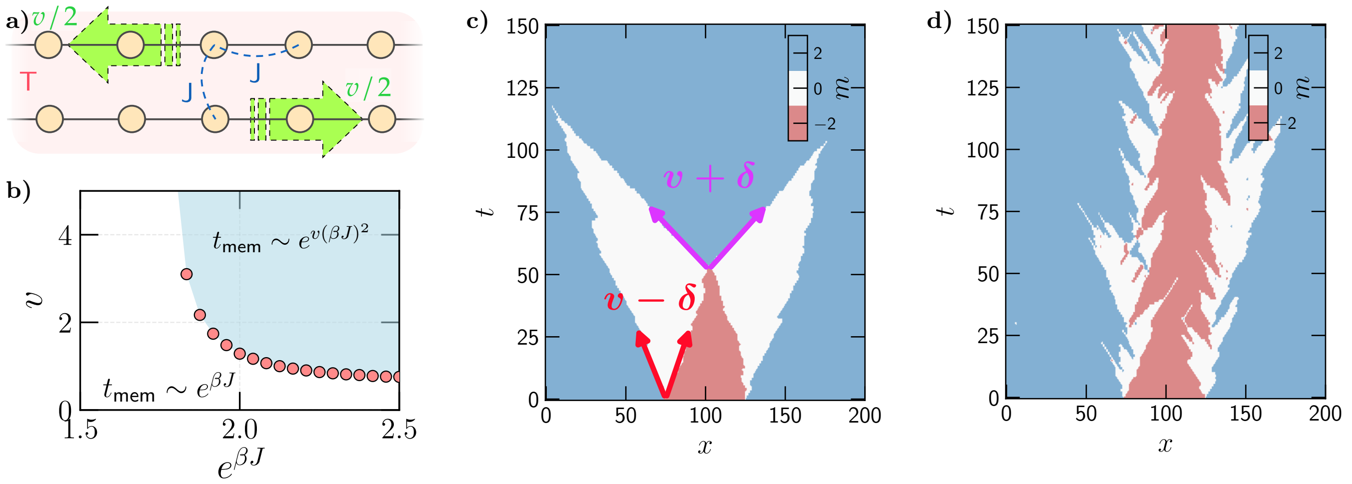

FIG. 1. **a)** Schematic of the dynamics. Two ferromagnetically coupled ferromagnetic 1d Ising chains with coupling J slide past one another, each at velocity v, and both couple to a bath at temperature T. **b)** Scaling of the memory time in the $e^{\beta J}$-v plane. In the white region, the memory time scales like $e^{O(\beta J)}$, just as in the equilibrium 1d Ising model. In the blue region, the memory time is enhanced to $e^{O(v(\beta J)^2)}$. The pink points mark the crossover between the two, and are drawn by fixing $e^{\beta J}$, and then finding the smallest value of v such that $t_{\mathsf{mem}}(v) > 10 t_{\mathsf{mem}}(0)$ (with the constant 10 chosen rather arbitrarily). **c)** A spacetime history of the magnetization, with $\beta = \infty, v = 1$. The indicated purple and red arrows denote interfaces that move with expected speeds $v + \delta$ and $v - \delta$, respectively. **d)** A history with $\beta = 2, v = 1$, with the minority domain in the initial state having size larger than ξ_{er}, the length scale below which minority domains are ballistically eroded.

where $E_C(t)$ is the energy of configuration C under $H(t)$, and C' differs from C by a single spin flip. In the following, we will fix the units of time so that $\gamma = 1$ [19]. Models of "magnetic friction" like this have been studied before [20–22], but past work has largely focused on the $v \to \infty$ limit, and has not investigated how sliding impacts memory times [23].

When $v = 0$, the dynamics is simply that of a static two-leg Ising chain, and $t_{\mathsf{mem}} \sim e^{O(\beta J)}$. The main result of this work is to show that t_{mem} is parametrically increased when v is greater than a constant threshold. In particular, we give analytic arguments—and verify in numerics—that as long as $v > v_c$,

$$t_{\mathsf{mem}} \sim e^{4(\beta J)^2 v + O(\beta J)}, \tag{4}$$

where the $\sim$ hides constant factors (see Fig. 1 **b** for a schematic phase diagram). We will argue below that $v_c \leq 1$ at large enough βJ.

While t_{mem} is finite in the thermodynamic limit for all $T > 0$, it rapidly gets astronomically large as T is decreased. For example, when $T = 0.6J$ and $v = 2.5$, we find numerically that $t_{\mathsf{mem}} \sim 10^{28}$, around 21 orders of magnitude larger than when $v = 0$ at the same temperature [24] (see below for more details). This enhanced lifetime does not depend on the $\mathbb{Z}_2$ spin-flip symmetry in (2), and breaking it with a small magnetic field h simply results in the replacement $J \mapsto J - |h|$. The remainder of this paper is dedicated to explaining this result.

The shearing mechanism: In the equilibrium 1d Ising model, once minority domains are created, there is nothing that compels them to shrink. Instead, they expand diffusively, and quickly erase memory of the initial magnetization. To understand why sliding changes this, consider what happens to minority domains under the sliding dynamics. Due to the interchain coupling, a minority domain on only a *single* chain is eroded ballistically from its endpoints, which violate two ferromagnetic bonds. We will write δ for the velocity at which the endpoints of a minority domain on a single chain contract; at small T the Glauber transition rates give $\delta \approx 1/(1 + e^{-2\beta J})$. Note that single-chain minority domains are eroded ballistically even in equilibrium, when $v = 0$.

A minority domain that spans *both* chains is more problematic. When $v = 0$, its endpoints diffuse, and nothing in the dynamics compels it to shrink. When $v > 0$ however, the sliding dynamics attempts to shear the domain, ripping it apart into two spatially-separated single-chain minority domains. The ferromagnetic interchain coupling attempts to arrest this shearing, as it favors configurations where domains on each chain "stick" together. But if $v > \delta$ and T is low enough, it is easy to check that the domains are sheared apart faster than they stick together, and once sheared apart, they are ballistically shrunk by the mechanism discussed above. This process, viz. the ripping apart of a two-chain minority domain and the subsequent ballistic erosion of its two constituent single-chain halves, is shown in Fig. 1 **c**, and is what leads to the aforementioned increase in t_{mem}.

In passing, we remark that this shearing mechanism produces dynamics that are qualitatively similar to Toom's two-line-voting automaton [12, 13] and the GKL automaton [3], two noisy cellular automata which were originally proposed as candidates for one-dimensional memories. This was shown not to be the case in [13], which proved that dynamics subjected to $\mathbb{Z}_2$-symmetry-breaking noise of strength ε has $t_{\mathsf{mem}} = e^{\Theta(\log^2(\varepsilon))}$ (it was conjectured from numerics in [25] that $t_{\mathsf{mem}} = e^{\Theta(1/\varepsilon)}$ for symmetric noise, but our analysis will show that in fact the $e^{\log^2(\varepsilon)}$ scaling holds in this case as well; see [26] for details).

How effective is the shearing mechanism? First consider what happens when $v \to \infty$: here, domains are sheared apart instantly, and a spin on one chain couples to spins on the other chain in a random order. The inter-chain interactions effectively become all-to-all, and the model enters a mean field limit, where it is easy to show the existence of a phase with long range order (see e.g. [20, 21]). In this regime, a system of size L has genuine long-range order, with $\lim_{L\to\infty} t_{\mathsf{mem}} = \infty$ at low enough T (and in fact we only need $v = \Omega(\log L)$ for this to be true).

At finite $v > \delta$, we will argue that t_{mem} scales as in (4) at low enough T (at small T, $\delta \leq 1$ thus sets an upper bound on the parameter v_c that controls when (4) holds). This argument will proceed in the following stages. First, we will show that two-chain minority domains (referred to as just "minority domains" in the following) are sheared apart with high probability as long as their size l satisfies $l < \xi_{\mathsf{er}}$, where the *erosion length* ξ_{er} is a length scale we will show behaves as $\xi_{\mathsf{er}} \sim v e^{\beta J}$. We will also show that minority domains of size $l > \xi_{\mathsf{er}}$ are *not* ballistically eroded, and that their endpoints instead undergo diffusive motion, as is the case for domains in the equilibrium 1d Ising model (see Fig. 1 **d** for an example). t_{mem} is thus set by the time it takes for the dynamics to spontaneously nucleate a minority domain of size ξ_{er}. We will show (4) by showing that, once a minority domain of a small constant size is created, the probability that it expands to size ξ_{er} scales as $e^{O(\beta J \ln \xi_{\mathsf{er}})}$. Finally, we will then check in numerics that both of the steps in this argument are correct.

The erosion length: We begin by establishing the scaling of the erosion length ξ_{er}. Fig. 2 shows a schematic of how minority domains are eroded in the $\beta J \to \infty$ limit. The white triangles are spacetime regions with $m = 0$ where the initial domain has been sheared apart, with minority spins present on only one of the chains. If a majority spin flips to match a minority spin in this region—which happens with probability $\sim e^{-2\beta J}$—it can rapidly nucleate a new domain with minority spins on both chains. These processes are what prevent the erosion of minority domains of sufficiently large size l, as when l is large enough, a majority spin flip occurring somewhere in this spacetime region is a likely event. When these events are likely, they result in the minority domain endpoints executing diffusive random walks, as exhibited in Fig. 1 **d**.

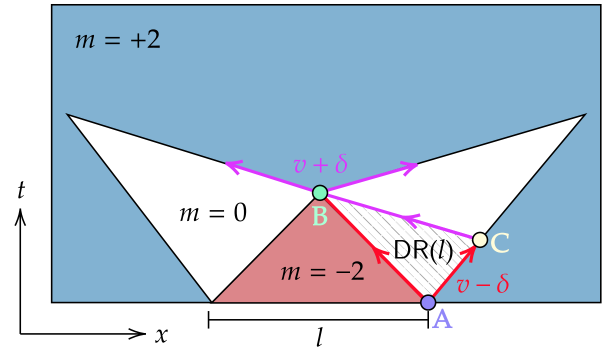

FIG. 2. How a minority domain of size l is eroded under ideal sliding dynamics with $v > 1$ in the limit $\beta J \to \infty$. The white regions have zero magnetization, and are metastable. The red arrows indicate interfaces that move with expected speed $v - \delta$, and the purple arrows those with speed $v + \delta$. The danger region $\mathsf{DR}(l)$ indicated by the shaded triangle is the spacetime region where spin flips in the metastable zero-magnetization area can propagate to prevent the erosion of the $m = -2$ (red) part of the initial minority domain.

Let us more carefully consider the spacetime region where events that flip majority spins can lead to the minority domain directly increasing in size. Consultation of Fig. 2 shows that this happens in the hatched spacetime region marked as $\mathsf{DR}(l)$, which we will refer to as a *danger region*. ξ_{er} can be estimated as the size l where having a majority spin flip somewhere in $\mathsf{DR}(l)$ becomes likely:

$$e^{-2\beta J} |\mathsf{DR}(\xi_{\mathsf{er}})| \sim 1. \tag{5}$$

$|\mathsf{DR}(l)|$ can be determined from simple geometric considerations as $|\mathsf{DR}(l)| \approx l^2 \delta/(4v(v-\delta))$ [26]. From (5), we then have

$$\xi_{\mathsf{er}} = c v e^{\beta J}, \tag{6}$$

where c is a constant independent of v to leading order in δ/v.

To determine t_{mem}, we thus need to understand the most likely process by which fluctuations produce minority droplets of size ξ_{er}. If such droplets had to be created "all at once" by rare fluctuations, or had to be built up continuously from size 1 to size ξ_{er}, we would expect $t_{\mathsf{mem}} \sim (e^{O(\beta J)})^{\xi_{\mathsf{er}}}$. This estimate is too naive, however, as there is a more efficient way of building up minority domains.

To understand this mechanism, consult again the schematic in Fig. 2. If a majority spin flips in DR, the largest amount by which it can increase the size of the minority domain is $x_C - x_A$, where $x_{A/C}$ are the spatial coordinates of the points $\mathbf{A}, \mathbf{C}$ in the figure. Basic geometric considerations show [26] that this distance equals $l\delta/2v$. A spin flip event with likelihood

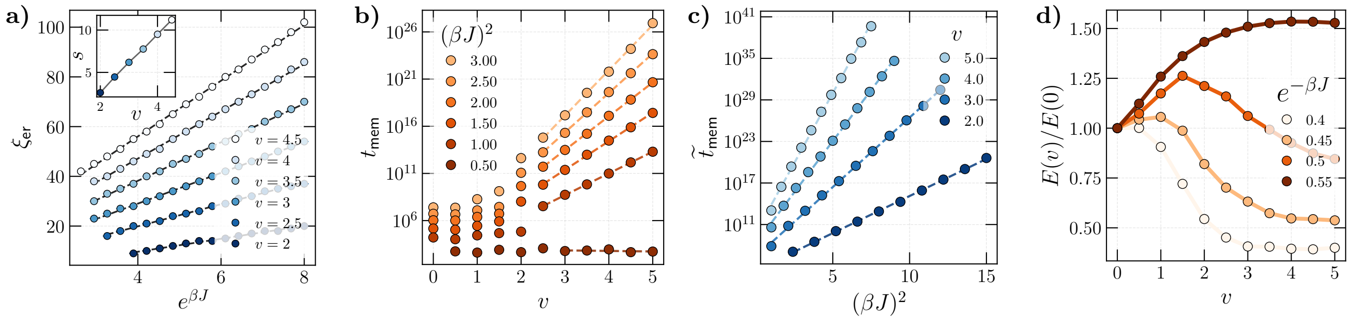

FIG. 3. **a)** The erosion length ξ_{er} against $e^{\beta J}$ for different values of v. The inset shows the slopes s of linear fits of ξ_{er} to $e^{\beta J}$ (drawn as dashed black lines), confirming that $\xi_{\text{er}} \sim v e^{\beta J}$. **b)** t_{mem} computed with forward flux sampling as a function of v, confirming that t_{mem} scales exponentially with v when $v > 1$ and βJ is large enough. Dashed lines are linear fits of $\ln t_{\text{mem}}$ to v in the range $v > 1$. **c)** Nucleationless memory time $\widetilde{t}_{\text{mem}}$ (see the main text) as a function of $(\beta J)^2$, indicating that t_{mem} indeed scales exponentially with $(\beta J)^2$ when $v > 1$ and βJ is large enough. Dashed lines are linear fits of $\ln \widetilde{t}_{\text{mem}}$ to $(\beta J)^2$. **d)** Average energy $E(v)$ in the steady state of the sliding dynamics at speed v, normalized to the value $E(0)$ of the energy at $v = 0$. At large enough βJ and v, the energy *decreases* with v. In all plots, standard error bars are smaller than the data points.

$\sim e^{-2\beta J}$ can therefore increase the size of the minority domain by at most $l \to l(1 + \delta/2v)$. Since the domain size increases multiplicatively, the number $n_{\xi_{\text{er}}}$ of such spin flips needed to grow the domain to size ξ_{er} is $n_{\xi_{\text{er}}} = \ln(\xi_{\text{er}})/\ln(1 + \delta/2v)$. We therefore expect

$$t_{\text{mem}} \gtrsim b(e^{2\beta J})^{\frac{\ln(\xi_{\text{er}})}{\ln(1+\delta/2v)}} \sim b e^{2(\beta J)^2 \frac{1}{\ln(1+\delta/2v)}}, \qquad (7)$$

for some constant $b > 0$ (dropping the $\ln(v)$ part of $\ln(\xi_{\text{er}})$). We have written $\gtrsim$ here because this argument ignores the time needed to nucleate a small minority domain that can then be expanded by the mechanism above, a time which scales as $e^{O(\beta J)}$. Adding this in and expanding in $\delta/v \ll 1$ then gives the main result (4) [27].

The above argument has taken place in the presence of a $\mathbb{Z}_2$ symmetry, brought about by the lack of a symmetry-breaking magnetic field. The sliding-induced enhancement of t_{mem} is however not dependent on this symmetry, and repeating the arguments above in the presence of a magnetic field simply results in the replacement $J \to J - |h|$ in (4).

Numerics: We now confirm these predictions in numerics, sticking for concreteness to the case with no magnetic field. We start with our prediction for ξ_{er}, which we estimate numerically as follows. First, we prepare an initial state containing a minority domain of length l in a system of size $L = 5vl$. We then evolve for a time l, and define the *erosion probability* $\mathsf{P}_{\text{er}}(l)$ as the probability that the total number of minority spins has decreased to less than l by this time. We then estimate ξ_{er} as the largest value of l where erosion happens with a probability greater than an arbitrary fixed constant, which we choose to be $3/4$ for concreteness:

$$\xi_{\text{er}} = \max\{l \,:\, \mathsf{P}_{\text{er}}(l) > 3/4\}. \qquad (8)$$

Fig. 3 **a** shows the result of measuring (8) in numerics, and confirms the predicted scaling (6) at large enough $\beta J, v$.

We now confirm that t_{mem} indeed scales as in (4). Doing so is infeasible with direct Monte Carlo updates, since the estimated values of t_{mem} quickly become extremely large. Fortunately, forward flux sampling (FFS) [28]—a technique that selectively isolates updates that move the magnetization out of its metastable minimum—allows us to get reliable estimates for t_{mem}, even in this setting. For details about FFS and the full codebase used to perform the numerics, see [26].

Fig. 3 **b** shows the FFS estimate of t_{mem} plotted against v for different values of βJ. As expected, we observe t_{mem} to grow exponentially in v when $v > 1$ (and perhaps at still smaller values of v). Confirming the scaling of t_{mem} with βJ is slightly more subtle. As discussed above, the seeding event where a small minority domain is nucleated contributes a factor of $e^{c\beta J}$ to t_{mem}. Creating a minority domain of just size 1 already gives $c = 8$ (the first flipped spin costs energy $\Delta E = 6J$, and the second costs $\Delta E = 2J$). Since the $e^{c\beta J}$ factor can compete with the $e^{O((\beta J)^2)}$ factor at modest values of βJ, we will find it convenient to get rid of the former by considering dynamics where the nucleation of an initial minority domain is not rare. We do this by modifying the dynamics to instantly insert a minority domain of size 2 whenever the system reaches a state with maximal or minimal magnetization. We write $\widetilde{t}_{\text{mem}}$ for the memory time under this dynamics, and plot $\widetilde{t}_{\text{mem}}$ against $(\beta J)^2$ in Fig. 3 **c**. The data are consistent with the $e^{O((\beta J)^2)}$ scaling of (4) as long as $\beta J \gtrsim \sqrt{2}$, and linear fits of $\ln \widetilde{t}_{\text{mem}}$ to $(\beta J)^a$ have lower R^2s for $a = 2$ than $a = 1$ for all values of v (see [26] for details).

Frictional cooling: How should $E(v)$, the expected value of $H(t)$ in the nonequilibrium steady-state at shearing velocity v [29], depend on v? Shearing minority domains apart temporarily leads to increases in energy, but if the sliding dynamics produces on net a smaller number of minority domains, it has a chance to nevertheless lead to a smaller value of $E(v)$. We show the result of measuring $E(v)$ numerically in Fig. 3 **d**, which shows that as long as βJ and v are large enough, $E(v)/E(0)$ decreases as a function of v. Usually, frictional forces between two sliding systems heat these systems up, and this is indeed what happens in our system at small $\beta J, v$. However, in the regime where sliding increases t_{mem}, the opposite is true: at least as judged by the steady-state energy, frictional forces instead cool the system down (see also the discussion in [20]).

Outlook: This work has shown that a very simple nonequilibrium drive—sliding two systems past one another at speed v—can dramatically slow down relaxation in a locally-interacting 1d spin system, increasing the memory time from $e^{O(\beta J)}$ at $v = 0$ to $e^{O(v(\beta J)^2)}$ at $v > v_c$, where $v_c \leq 1$. This increase happens because the critical size ξ_{er} of a minority domain that is not eroded by the dynamics increases from $\xi_{\mathsf{er}} = 1$ at $v = 0$ to $\xi_{\mathsf{er}} \sim ve^{\beta J}$ at large enough $\beta J, v$.

How slow can we go? Two things in this model limit how large t_{mem} can get. The first is the scaling of ξ_{er} with βJ: is it possible to design models where ξ_{er} grows even faster? Additionally, in our model, the creation of a critical domain is suppressed in probability only by a factor of $e^{-O(\beta J \ln \xi_{\mathsf{er}})}$: are there models where this suppression instead scales as $e^{-O(\beta J \xi_{\mathsf{er}})}$? Answering either of these questions in the affirmative would move us even closer to constructing a simple model with long-range order in a locally-interacting 1d system. Beyond this, it will also be interesting to examine other types of sliding in two and higher dimensions, where sliding is known to modify critical points [21, 30], and to consider generalizations with different symmetry groups [31]. Finally, it is natural to wonder what other nonequilibrium phenomena can be realized in this setting. Reference [11] recently constructed simple nonequilibrium drives that can simulate *arbitrary* types of active dynamics, and it will be interesting to understand if similar results hold for sliding and related mechanisms.

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[19] In typical magnetic systems, the time for a local moment to relax to its Weiss field is around 10^{-8}s [30]. Thus in our units, if we use a lattice spacing of 10^{-10}m, a velocity of $v = 1$ corresponds to a dimensionful speed of around $1\,\mathrm{cm\,s^{-1}}$.

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